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Financing and Advising: Optimal Financial Contracts with Venture Capitalists

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Abstract

This paper offers a rationale for the joint provision of funds and advice by financiers, such as venture capitalists. I consider a double moral hazard context where the entrepreneur and the advisor can exert unobservable efforts to enhance productivity. Without moral hazard, it is optimal that both exert effort. With moral hazard, if the entrepreneur's effort is more efficient than the advisor's effort, the latter is not hired if she does not provide funds. Financial partnership between the entrepreneur and the advisor is thus derived endogenously. I show how it can be implemented with convertible bonds or preferred stocks and discuss how the results of the model fit the stylized facts on venture capital contracts.

keywords: venture capital, convertible bonds, preferred stocks, double moral hazard.

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1 Introduction

The venture capital industry has grown dramatically over the last decade. In the US, venture capital (hereafter VC) investments grew from \$ 3.3 billion in 1990 to \$ 100 billion in 2000. In Europe funds invested in VC grew from \$ 6.4 billion in 1998 to more than \$ 10 billion in 1999. At the origin of this success is certainly the value-added by these investors. Sahlman (1988, 1990) reports that venture capitalists spend a lot of time selecting projects and remain deeply involved in the post-investment development of those projects. This involvement gives them the opportunity to extract information on the quality of their investment (Gompers (1995)), to monitor and directly control the entrepreneurs (Lerner (1995), Hellmann and Puri (2001)), and also to provide managerial advice to those entrepreneurs. The importance of this advising role has been extensively documented empirically[1]. Venture capitalists contribute to defining the firm's strategy and financial policy, professionalizing their internal organization, and recruiting key employees.

This paper provides a theory of start-up financing based on the dual, financing and advising, role played by venture capitalists. Entrepreneurs endowed with the creativity and technical skills needed to develop innovative ideas may lack business expertise and need managerial advice. I analyze a model where, in the first best, some effort should be provided both by the entrepreneur and by the advisor. In line with the view that entrepreneurial vision is really key to the success of the venture, I assume the entrepreneur's effort is more efficient than the advisor's. I consider the case where advice can be provided by consultants or by venture capitalists. Quite plausibly, I assume the level of effort exerted by the advisor to develop the project, as well as by the entrepreneur, are not observable. Consequently the entrepreneur and the advisor face a double moral hazard problem. In order to induce them to support the firm's project and enhance the profitability of the venture, both the entrepreneur and the advisor must be given proper incentives through the cash-flow rights they receive over the outcome of the project. In addition to effort, the project requires financial investment. This can be provided by the entrepreneur, the advisor or pure financiers.

The first question raised in the paper is the following : why should the entrepreneur ask for advice from venture capitalists rather than from consultants? What makes VC advising different from

[1] See Gorman and Sahlman (1989), Sahlman (1990), Bygrave and Timmons (1992), Gompers and Lerner (1999), and more recently Hellmann and Puri (2001).

consultant advising ? I show that, even if the entrepreneur is not wealth-constrained, and could fund himself all the initial investment, he chooses to obtain funding from the advisor, thus relying on VC advising rather than on consultants[2]. To understand the intuition of the result, consider the extreme case where the advisor could not provide funds. In this case, although the project would be more profitable with external advice, the entrepreneur chooses not to hire a consultant. This is because the rent the entrepreneur would need to leave to the consultant (to incentivize her) is too high. If, in contrast with our maintained hypothesis, the advisor's effort was more efficient than the manager's, (pure) consultants could be hired in equilibrium. This suggests that the relative roles of consultants and venture capitalists depend on how crucial their advice is to the success of the ventures. More drastic innovations that rely on the entrepreneur's human capital are more likely to seek for VC advising rather than consultant advising.

The second question raised in the paper concerns the implementation of the contract between the entrepreneur and the venture capitalist. The way the financial agreement is designed must take into account the two agents' incentives. It must also provide them an expected return at least equal to their investment. Consequently, two regimes arise depending on the amount invested by the investor. When the amount invested by the venture capitalist is low, he receives common stocks, while the entrepreneur is given preferred equity. When the amount invested by the venture capitalist is high, he is given convertible bonds or preferred equity. The intuition of this result is that when the investment of one agent is low, she gets a small share of outcome. In order to incentivize her, she must be given higher powered incentives. In the first regime, the investor is given more powerful incentives to exert effort (i.e. her marginal revenue is higher in high states than in low states) because her investment is low. The second regime corresponds to the symmetric case, where the entrepreneur must be given higher powered incentives, since his investment is lower.

These results provide a possible explanation for the link between investment and financial contracts set by start-ups and VC investors. Fenn, Liang and Prowse (1998) observe that business angels tend to invest smaller amounts of money than venture capitalists, and to acquire common stocks. Venture capitalists however tend to acquire convertible bonds (see also Kaplan and Strömberg (2000)). The two regimes can be interpreted as respectively business angel financing and venture capitalist financing.

[2] Of course, when the entrepreneur is wealth-constrained, VC financing is all the more desirable.

The present model can thus be viewed as a first step towards understanding the differences between business angels and venture capitalists. While both of these types of investors play a significant role in early stage financing, the analysis of their differences has not received, to our knowledge, much attention in the literature so far.

Last, the model provides testable implications on the link between the level of investment, the project's profitability and the type of financial claim that is issued. Indeed, the first regime is more likely to arise when the initial outlay is low compared to the expected cash-flows of the project while the second regime illustrates the case where the initial outlay is relatively high compared to the expected cash-flows of the project and compared to the wealth of the entrepreneur.

The present model offers a rationale for the use of convertible bonds or outside equity in the financing of start-ups, to incentivize the investor *and advisor*.[3] Other papers explained the use of convertible claims in VC financing by focusing on the incentives convertible claims provide to managers[4]. For example, Green (1984) and Biais and Casamatta (1999) show that convertible bonds induce managers to exert effort while precluding inefficient risk-taking.

While the current paper focuses on how VC contracts deal with moral hazard issues, and provide incentives to exert effort to increase the probability of success of the venture, Bergemann and Hege (1998), Cornelli and Yosha (1997), Dessì (2001) and Habib and Johnsen (2000) analyze how financial contracts elicit information revelation, and are useful to discriminate across projects and take efficient continuation or liquidation decisions[5].

The special focus of our model on the efficiency of the joint efforts of the manager and the investor is shared by a couple of recent papers. In Repullo and Suarez (1999), unlike in the present paper, the entrepreneur doesn't have the option to implement the project alone. This makes our first question irrelevant in their setting. Schmidt (1999) also considers a double moral hazard setting to explain the

[3]An original approach is developped in Cestone and White (1998) who find that outside equity acts as a commitment device for the venture capitalist not to fund competing firms.

[4]To the extent that the model derives the optimality of a mix of outside debt and outside equity, it is also related to Chang (1993), Fluck (1998) or Dewatripont and Tirole (1994) which do not specifically focus on venture capital finance.

[5]Admati and Pfleiderer (1994) first studied the problem of acquisition of information in the context of stage financing. They argue that assigning a fixed claim to the venture capitalist prevents him from strategic trading and induces optimal continuation decisions.

use of convertible bonds in VC financing. However, investment in his model is an unobservable variable, while our model distinguishes between financial investment and effort. In contrast with these papers, our model endogenizes the level of financial investment by the venture capitalist, and explains under which conditions consultants are not valuable for the entrepreneur. In our analysis optimal financial claims have a double purpose : ensuring that each party will recoup her investment while maintaining incentives to effort.

The present model analyzes the consequences of moral hazard, due to unobservable effort, and studies how the allocation of cash flow rights enables one to cope with this problem. Another branch of the literature takes an alternative route, by emphasizing contract incompleteness, due to observable but non contractible decisions, and analyzes in this context the allocation of different types of control rights between managers and investors (Chan, Siegel and Thakor (1990), Berglöf (1994), Hellmann (1998), Kalai and Zender (1997), or Cestone (2000)). The empirical study conducted by Kaplan and Strömberg (2000) show that both the allocation of cash-flows and the allocation of control rights play an important role in practice. In the present paper, for clarity, we focus on only one of those two aspects.

The paper is organized as follows : the model and the assumptions are presented in section 2. Section 3 solves for the optimal contract. It studies why entrepreneurs are unwilling to hire consultants and derives the characteristics of the contracts between entrepreneurs, financiers and venture capitalists. Section 4 discusses how to implement the contracts between the VC and the entrepreneur with financial claims such as convertible bonds or stocks. Concluding remarks are proposed in section 5. All proofs are in Appendix.

2 The model

Consider an entrepreneur endowed with an innovative investment project. The project requires three types of inputs : one contractible initial investment I (money) and two unobservable (hence non contractible) investments denoted e and a, where e represents the innovative effort put into the project and a the management effort to run the project properly. The project is risky and generates a verifiable random outcome $\widetilde{R}$. To keep things simple, assume that it can either succeed or fail. Hence, $\widetilde{R}$ can take two values : R^u in case of success, and R^d ($< R^u$) in case of failure. The probability of success is denoted p_u ($(1 - p_u)$ is thus the probability of failure).

The production technology is the following : if I is not invested, p_u is equal to 0 ; if I is invested, $p_u = \min[e + a; 1]$[6], where e and a are continuous variables that take values between 0 and 1.

There is also a continuum of risk-neutral advisors and pure financiers. The different types of agents differ by their ability to provide the non contractible efforts e and a. Specifically, e can only be provided by the entrepreneur. This reflects the fact that the project is innovative and specific to the entrepreneur. For instance, it may consist in launching a new product or service, based on the idea developped by the entrepreneur himself, requiring his personal expertise. However, a cannot be provided by the entrepreneur. Although the entrepreneur is endowed with the technical skills and creativity required to develop his idea, he is an absolute beginner in business administration, and lacks the expertise to build a business plan, define the firm's marketing strategy... By assumption, a can be provided by an advisor, who is supposed to have business experience. Pure financiers can't provide a or e.

Both efforts are costly. Let $c_E(.)$ denote the entrepreneur's disutility of effort, and $c_A(.)$ the advisor's disutility of effort. By assumption, we have :

$$c_E(e) = \beta \frac{e^2}{2} \text{ and } c_A(a) = \gamma \frac{a^2}{2}.$$

Moreover, for a given level of effort, the cost is lower for the entrepreneur than for the advisor : $\gamma > \beta$, i.e., the effort of the entrepreneur is more efficient. It would be equivalent to consider that the two agents have the same cost function, and that the impact of each effort on p_u is weighted by $\frac{1}{\beta}$ and $\frac{1}{\gamma}$ respectively. Intuitively, this assumption captures the idea that the insights and vision of the entrepreneur are more important for success than the managerial expertise of the advisor.

Agents are not a priori wealth-constrained, that is, any of them can provide (part of) the initial investment I. However we will assume that once the firm is created, agents are protected by limited

[6] The assumption of unobservable effort to increase the probability of success of the project is taken from Holmstrom and Tirole (1997). Note that the structure of probability chosen implies that the two efforts are not complementary : their joint realization is *not* required to implement the project. Instead, each effort contributes to improve the profitability of the project.

liability (i.e., the only thing that can be shared is the outcome of the project)[7]. As all agents are risk-neutral, their opportunity cost of putting money into the firm is the riskless interest rate r, normalized to zero. Denote A_{VC} the amount of money provided by the advisor, A_F the money provided by the pure financier, and $I - A_{VC} - A_F$ the money provided by the entrepreneur[8]. If $A_{VC} = 0$, then the advisor who exerts effort a will be called a consultant, while if $A_{VC} > 0$, she will be called a venture capitalist.

Accordingly, the social value of the project is :

$$V(e,a) = \min[e+a;1]R^u + \max[0;1-(e+a)]R^d - \beta\frac{e^2}{2} - \gamma\frac{a^2}{2} - I. \tag{1}$$

As a benchmark, let's determine the optimal levels of efforts when all inputs are contractible (i.e. when efforts are observable). This corresponds to the first-best solution that maximizes the social value of the project. It is straightforward to see that it is optimal to have both the entrepreneur and the advisor exert strictly positive levels of effort. Indeed, when $p_u < 1$, we have :

$$\left.\frac{\partial V}{\partial e}\right|_{e=0} = R^u - R^d > 0,$$

and

$$\left.\frac{\partial V}{\partial a}\right|_{a=0} = R^u - R^d > 0.$$

When both efforts are observable, the optimal levels of effort are given by the first order conditions of the maximization of V:

$$e^{FB} = \frac{1}{\beta}(R^u - R^d),$$

and

$$a^{FB} = \frac{1}{\gamma}(R^u - R^d).$$

For our notations to be consistent, let's assume : $\left(\frac{1}{\beta}+\frac{1}{\gamma}\right)(R^u - R^d) < 1$, so that the constraint $\min[e+a;1] \leq 1$ is never binding. Note that as the entrepreneur's effort is more efficient (or less costly) than the effort of the advisor, the optimal level of effort e^{FB} is larger than a^{FB}.

[7]This assumption is in the line of Innes (1990) and is meant to make the problem interesting under risk neutrality.

[8]Note that the amount of money the entrepreneur puts into the firm may be negative if $A_{VC} + A_F > I$, in which case he receives a strictly positive transfer when investment is made.

This first best solution can be implemented in a number of ways. Specific investments e and a must of course be provided by the entrepreneur and by the advisor respectively. But the identity of the agent providing the financial investment I is irrelevant. Participation is ensured to the extent that the agent which investment is needed receives an expected outcome at least equal to the opportunity cost of her investment[9]. In other words, what matters to implement the first-best is how much each party will earn in expectations, not the way the outcome is shared (this, of course, won't be true anymore when efforts are not observable : the way the outcome is shared will determine how much effort will be provided). For instance, a possible solution is to write a contract by which a consultant provides an effort level a^{FB}, a financier provides a financial investment I, and the entrepreneur provides an effort level e^{FB}. The consultant and the financier will agree as long as they are promised an expected outcome of respectively $c_A(a^{FB})$ and I. An alternative solution would be to ask a venture capitalist to provide a^{FB} and I, and to promise her an expected outcome of $c_A(a^{FB}) + I$.

The first-best value of the project is then given by :

$$V^{FB} = \frac{1}{2}\left(\frac{1}{\beta} + \frac{1}{\gamma}\right)(R^u - R^d)^2 + R^d - I.$$

When there is no moral hazard problem, it is always optimal for the entrepreneur to ask for the services of an advisor. Whether the advisor is a consultant or a venture capitalist is irrelevant : the same social value can be attained when a financier, an advisor or the entrepreneur himself provides the financial investment I. We will see later that this contrasts sharply with the conclusions derived under moral hazard.

3 Optimal contract with moral hazard

When the efforts are not observable, they are chosen by each agent maximizing her expected utility. As all agents are risk-neutral, their expected utility is perfectly identified by their net expected gains. Those gains are determined by the financial contract they agree on, which specifies :

- the financial investment to be made by each party,
- the share of the revenue allocated to each party in each state of nature.

[9]This is always feasible since by assumption the NPV of the project is strictly positive in the first best.

Denote α_E^θ (resp. α_A^θ) the share of the revenue accruing to the entrepreneur (resp. the advisor) in state $\theta \in \{u, d\}$. If a pure financier is included in the contract, she receives a share : $1 - \left(\alpha_E^\theta + \alpha_A^\theta\right)$ in state θ.

The level of effort chosen by the entrepreneur is given by his incentive compatibility condition, denoted $(IC)_E$:

$$e \ \in \arg\max_{\widehat{e}} (\widehat{e} + a)\alpha_E^u R^u + (1 - (\widehat{e} + a))\, \alpha_E^d R^d - \beta \frac{\widehat{e}^2}{2} - (I - (A_{VC} + A_F)),$$

which means that he chooses the level of effort that maximizes his expected profit, given the contract established and his cost of effort.

Equivalently, the incentive compatibility condition of the advisor, denoted $(IC)_{VC}$, is given by :

$$a \ \in \arg\max_{\widehat{a}} (e + \widehat{a})\alpha_A^u R^u + (1 - (e + \widehat{a}))\, \alpha_A^d R^d - \gamma \frac{\widehat{a}^2}{2} - A_{VC}.$$

Last, assume that the financial contract maximizes the expected utility of the entrepreneur. This reflects the assumption that the entrepreneur has a unique, innovative idea, but can ask for business advice and money from a large number of agents. The participation constraints of the advisor and the financier, assuring that they recoup at least their investment (in expectations), must be added. The participation constraint of the advisor, denoted $(PC)_{VC}$, is :

$$(e + a)\alpha_A^u R^u + (1 - (e + a))\, \alpha_A^d R^d - \gamma \frac{a^2}{2} \geq A_{VC}.$$

The participation constraint of the financier, denoted $(PC)_F$, is :

$$(e + a)(1 - (\alpha_E^u + \alpha_A^u))R^u + (1 - (e + a))\, (1 - (\alpha_E^d + \alpha_A^d))R^d \geq A_F.$$

Hence the program to be maximized is :

$$\begin{aligned}
\max_{e, a, \alpha_E^\theta, \alpha_A^\theta, \theta \in \{u,d\}, A_{VC}, A_F} \quad & (e + a)\alpha_E^u R^u + (1 - (e + a))\, \alpha_E^d R^d - \beta \frac{e^2}{2} - (I - (A_{VC} + A_F)), \\
s.t. \quad & (PC)_{VC}, \\
& (PC)_F, \\
& (IC)_{VC}, \\
& (IC)_E,
\end{aligned}$$

$$(\alpha_E^u, \alpha_E^d, \alpha_A^u, \alpha_A^d) \geq 0$$

$$\alpha_E^u + \alpha_A^u \leq 1$$

$$\alpha_E^d + \alpha_A^d \leq 1,$$

where the last three conditions are feasibility constraints ensuring limited liability holds for all agents.

The following lemma states what levels of effort are chosen by the entrepreneur and by the advisor as a function of the parameters of the contract.

Lemma 1 *When the parameters of the model are such that $\frac{1}{\beta}R^u < 1$ (A.1), the levels of effort e and a are given by the first order conditions of the incentive compatibility constraints $(IC)_E$ and $(IC)_{VC}$:*

$$e = \frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d),$$

and

$$a = \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d).$$

For each agent, the level of effort increases in the difference between her profit in state u and her profit in state d. Indeed, e (resp. a) is increasing in α_E^u (resp. α_A^u), and decreasing in α_E^d (resp. α_A^d). Since both efforts are decreasing with the agents' revenue in the bad state, pledging the revenue in case of failure to a third party (here, the pure financier) could boost the two agents' incentives to provide effort. We will see later that this incentive effect gives scope for outside financing by a pure financier in this simple model, although there is no wealth constraint. Still, limited liability implies that the two agents must share a fixed final outcome in case of success. This means that the more powerful the incentives given to one agent, the less the other agent will be induced to provide effort. The optimal contract will take into account the countervailing effects of inducing one agent to work, and reflect both incentives. Last, note that assumption (A.1) simply insures that we get an interior solution when one agent is given maximal incentives. In the remaining of the analysis, $A.1$ will be assumed to hold.

3.1 What can be achieved with a pure consultant

The previous section established that without moral hazard problems, the entrepreneur was indifferent between hiring a consultant (i.e. asking for the services of an advisor without requiring her financing the firm) and contracting with a venture capitalist. This is because the first best solution can be attained in both cases, and the entrepreneur can capture the whole surplus under any financial agreement by simply rewarding each agent for his cost of investment. Under moral hazard, hiring a consultant (instead of a VC) will restrain both the net present value of the project *and* the part of it that the entrepreneur can get.

The following proposition establishes that the entrepreneur never chooses to hire a pure consultant.

Proposition 1 *Consider the case of a pure consultant, i.e. $A_{VC} = 0$. Then the entrepreneur is strictly better off without hiring a consultant. Moreover, the entrepreneur exerts the first best level of effort e^{FB} if the amount of outside financing is not too large in the sense that $A_F \leq R^d$.*

The intuition of proposition 1 is the following. To induce the consultant to exert effort, the entrepreneur needs to give her a strictly positive share of the final outcome in case of success. This affects the entrepreneur's own profit in three ways. The first one is a direct revenue effect : the entrepreneur's share of outcome is lower. The second one is an incentive effect : having a lower share of outcome, the effort provided by the entrepreneur decreases and this is not fully compensated by the effort exerted by the consultant (this is because the consultant's effort is less efficient) : overall, the probability of success decreases. The third effect is a reduction in the entrepreneur's cost of effort (since his effort is lower). The first two effects affect negatively the entrepreneur's profit while the third effect is positive. However the cost effect is not high enough to compensate the first two, and the entrepreneur maximizes his profit by not hiring a consultant. This of course is inefficient : because the costs of efforts are additive convex functions, it would be socially efficient to "split" the provision of effort between the two agents (even if one is less efficient). Starting from the case presented in proposition 1 where the entrepreneur does not hire an advisor, a small amount of business advice would increase the value of the project. The entrepreneur does not find this enhancement in social value efficient, however. Indeed, the rent he would have to leave to the consultant would be too large compared to the increase in value the consultant's advice would induce.

The main result of proposition 1 comes from the combination of two conditions. First, the consultant is less efficient, and second, the entrepreneur proposes the contract (hence he maximizes his own profit). If one of these assumptions is relaxed, it becomes optimal to hire a consultant. Consider for instance the case where a third party can decide whether to hire or not the entrepreneur and the consultant

: this third party would find it optimal to hire both agents eventhough one is more efficient than the other (because the joint provision of effort is value-enhancing) : the situation is then one of a standard moral hazard in team. Consider next the case where the agent who is less efficient (here the consultant) would have to decide whether or not to hire the other one (here the entrepreneur). Again she would find it optimal to hire the entrepreneur. In our model, however, the entrepreneur has a unique, specific idea which he can decide to implement alone. This allows him to choose not to hire a consultant. In the following section, we will see that one way to overcome this inefficiency is to ask the advisor to participate financially to the project, in the spirit of venture capital financing and advising. Intuitively, asking money to the advisor compensates the entrepreneur for granting the advisor a share of the proceeds and reduces the cost of getting business advice.

The last part of proposition 1 simply illustrates that the first best level of effort is achieved when the entrepreneur can get the marginal benefit of his effort. Indeed, if A_{VC} is lower than R^d, the revenue promised to the financier is a constant (i.e. it doesn't depend on the outcome), and the entrepreneur captures any increase in value induced by his effort. This is reminiscent of the classical Harris and Raviv (1979) result. However, due to limited liability, if outside financing is higher than R^d, the first best level of effort cannot be achieved anymore : the level of effort provided by the entrepreneur decreases.

3.2 Properties of the optimal contract when all agents can invest

We now turn to the case where all agents can invest money into the firm, that is A_{VC} and A_F can both be positive. It is easy to see that the two participation constraints PC_{VC} and PC_F must be binding. Then the program boils down to maximizing the NPV of the project subject to the incentive compatibility conditions and the feasibility conditions described at the beginning of this section. The next proposition presents the properties of the optimal contract between the entrepreneur, the VC and the pure financier.

Proposition 2 *When all agents can invest, it is optimal to set $A_F^* > 0$ and $A_{VC}^* > 0$. Moreover, the shares of outcome given to each party verify :*

- *if* $R^u \leq 2R^d$ *:*

$$\alpha_E^{u*}R^u - \alpha_E^{d*}R^d = \alpha_A^{u*}R^u - \alpha_A^{d*}R^d = R^u - R^d,$$

- *if* $R^u > 2R^d$ *:*

$$\begin{aligned} \alpha_E^{d*}R^d &= \alpha_A^{d*}R^d = 0, \\ \alpha_E^{u*}R^u &= \tfrac{\gamma R^d + \beta(R^u - R^d)}{\gamma+\beta}, \\ \alpha_A^{u*}R^u &= \tfrac{\beta R^d + \gamma(R^u - R^d)}{\gamma+\beta}, \end{aligned}$$

The first result of proposition 2 states that both the VC and the pure financier must invest strictly positive amounts of money into the project. The financial investment of the VC compensates the entrepreneur for conceding part of the outcome of the project to incentivize the VC. The financial investment of the pure financier allows to give her (part of) the revenue in case of failure, which relaxes the two incentive compatibility constraints. Using lemma 1, it is easy to see that when R^u is not very large compared to R^d in the sense defined in proposition 2, the presence of a pure financier induces the entrepreneur and the VC to exert the first best levels of effort e^{FB} and a^{FB}. This is because when the difference between the two possible outcomes is not very large, first best levels of effort are not very high, and it is possible to simply split R^u between the entrepreneur and the VC so that they exert e^{FB} and a^{FB}. This is no longer true when the difference between R^u and R^d is large, precisely because first best levels of effort are very high. In that case, the necessity to provide high-powered incentives to the two agents to exert effort requires to pledge the entire revenue R^d to the pure financier, and to let the entrepreneur and the VC share R^u in case of success. Still, the efforts provided by the two agents are strictly lower than in the first best. Note that if we are to model a new, highly risky project, it is very likely that the difference between R^u and R^d is large which means that in this model, the second case is more plausible.

The optimal contract described in proposition 2 exhibits features of the "live or die" contract first mentionned in Innes (1990). To provide maximal incentives to effort, it is optimal to pledge low states outcomes to the pure financier and let the agent get high states outcomes. In line with Innes' argument, this type of contract may not be sustainable for the following reason : the entrepreneur and the VC have an incentive to collude (by for instance injecting money into the firm) to claim state R^u

occurred while R^d actually occurred[10]. Avoiding such a collusive behavior requires to impose that the revenue of the pure financier doesn't decrease with the outcome of the project. This assumption will be maintained throughout the remaining of the analysis.

3.3 Optimal contract when all agents can invest : the case of non-decreasing revenue of the pure financier

In order to derive contracts robust to collusion, it is necessary to add the following constraint to the general program presented page 26 :

$$(1-(\alpha_E^u+\alpha_A^u))\,R^u \geq \left(1-(\alpha_E^d+\alpha_A^d)\right)R^d,$$

which ensures that the revenue of the pure financier doesn't decrease with the outcome of the project. As stated in the following proposition, the main difference with the analysis in the previous subsection is that the presence of a pure financier is no longer necessary.

Proposition 3 *With a non decreasing revenue constraint, it is optimal to set* $A_{VC}^* > 0$. A_F^* *can take any value in the interval* $[0; R^d]$. *Moreover, the shares of outcome given to each party verify :*

$$\begin{aligned}\alpha_E^{u*}R^u-\alpha_E^{d*}R^d &= \tfrac{\gamma}{\gamma+\beta}(R^u-R^d),\\ \alpha_A^{u*}R^u-\alpha_A^{d*}R^d &= \tfrac{\beta}{\gamma+\beta}(R^u-R^d).\end{aligned}$$

Proposition 3 implies that the levels of effort that can be achieved are independant from the financial participation of the pure financier. This is in sharp contrast with the results of proposition 2. Recall that the presence of a pure financier was found desirable to relax the two incentive compatibility constraints : pledging R^d to the financier in case of failure helps "punishing" the entrepreneur and the VC in the bad state (since their revenue is equal to zero) ; leaving R^u to the two agents who provide effort helps "rewarding" them in case of success. The non decreasing revenue constraint imposes to give to the financier at least as much in the good state as in the bad state. On the one hand, pledging (part of) R^d to the financier boosts incentives but on the other hand, it reduces the part of the good

[10] Such a situation may happen if, although the monetary outcome of the project is perfectly verifiable, the origin of this outcome is not.

outcome that can be distributed to the entrepreneur and the VC. The positive and negative effects end up cancelling each other, so that the presence of a pure financier becomes neutral[11].

Corollary 1 *The optimal amount of outside financing* ($A_F^* + A_{VC}^*$) *lies in the interval* $[K; K + R^d]$ *, where* K *is a positive constant defined in appendix. The maximal possible NPV of the project, denoted* V^**, is constant in this interval and strictly lower than* V^{FB}*.*

Corollary 1 is obtained by replacing the parameters of the contract by their value in proposition 3 into the participation constraints of the VC and the pure financier. Since the optimal levels of effort are constant in the interval $[K; K + R^d]$ and strictly lower than the first-best levels of effort, the second part of the corollary follows immediately. Not surprisingly, moral hazard problems lead to less efficient projects. Moreover, corollary 1 implies that the entrepreneur is always willing to have a financial partner investing in the project, eventhough he is wealthy enough to implement the project alone. When requiring the financial participation of the advisor, the entrepreneur makes her pay for the rent he has to leave to incentivize her : increasing the outside financing increases the ability of the entrepreneur to make the advisor work. As a consequence, the net present value of the project is higher, and so are the expected gains of the entrepreneur[12]. A real *financial partnership* with the advisor is thus desirable for the entrepreneur. Last, the corollary implies that, if the initial investment I that is needed to implement the project is larger than $K + R^d$, financial participation from the entrepreneur is also required to attain the optimum. It has been assumed so far that no agent is wealth-constrained, which means in particular that the entrepreneur is wealthy enough to complement if needed the financing of the project on top of the VC's and the financier's contributions. At this point, it is natural to relax this assumption and to investigate what can be achieved according to the wealth of the entrepreneur.

Proposition 4 *Suppose* $I > K+R^d$ *and the entrepreneur is not wealthy enough to invest* $I-(K+R^d)$*. Then the optimal contract is defined by :*

[11] Although the participation of a pure financier doesn't play an incentive role anymore, it may still be desirable if some agents are wealth-constrained since it can reduce the amount the other agents have to put into the project.

[12] As the participation constraints of the outside investors are always binding, the net expected profit of the entrepreneur is equal to the NPV of the project.

- *if* $A_F + A_{VC} \in]K + R^d; \overline{I}]$ *:*

$$\begin{aligned} \alpha_E^{d*} &= 0, \\ \alpha_E^{u*} R^u &= \tfrac{(\gamma-\beta)(R^u - R^d) + \gamma\beta\sqrt{\Delta}}{2\gamma-\beta}, \\ \alpha_A^{u*} R^u - \alpha_A^{d*} R^d &= \tfrac{\gamma(R^u - R^d) - \gamma\beta\sqrt{\Delta}}{(2\gamma-\beta)R^u}. \end{aligned}$$

- *if* $A_{VC} + A_F > \overline{I}$*, no parameter of the contract can satisfy the investors' participation constraints.*

$\overline{I}$ *is a constant and* Δ *is a function of the amount of outside financing* $A_F + A_{VC}$ *specified in appendix.*

Proposition 4 states that when the amount of outside financing is (suboptimally) high due to the entrepreneur being wealth-constrained, it is optimal to pledge R^d to the outside investors (VC and/or financier). The reason is that setting α_E^{d*} to zero relaxes the participation constraints while maintaining incentives to effort. Moreover, there is a maximal amount of outside financing the entrepreneur can raise. Above $\overline{I}$, too large a share of profits must be left to the investors so that they recoup their investment. This in turn destroys the entrepreneur's incentives to effort and leads to a negative NPV project : the participation constraints cannot be satisfied anymore.

Corollary 2 *When* $I > K + R^d$*, the entrepreneur's effort* e^* *decreases with the amount of outside financing. At the opposite, the VC's effort* a^* *increases with outside financing.*

Corollary 2 is simply the result of tightening the participation constraints of the outside investors. While the result is easily established using the formulas presented in proposition 4, the incentive effects at stake can be explained as follows.

Consider an initial financial arrangement whereby the pure financier invests $A_F = R^d$, the VC invests $A_{VC} = K$ and the entrepreneur invests the complement $I - (K + R^d)$. To ensure an expected outcome equal to R^d to the financier while preserving incentives to effort, it is optimal to pledge her the entire revenue in case of failure, and a revenue R^d in case of success. Corollary 1 states that the NPV is maximal in such a situation. Suppose now that the entrepreneur is wealth-constrained, so that the VC has to increase her financial participation[13]. The outcome to be given to the pure financier is

[13] Recall from proposition 3 that the maximal funding to be provided by the financier is R^d, hence she cannot increase her participation if the entrepreneur is wealth-constrained.

unchanged. To ensure that the VC recoups her investment, her share of outcome in case of success α_A^{u*} must increase, which decreases the share α_E^{u*} of the entrepreneur. As a consequence, the entrepreneur's effort decreases, and the VC's effort increases.
The same result applies if the financier increases her investment. Consider the initial situation where the VC invests $K + R^d$ and the entrepreneur invests the complement (hence, initially, the financier doesn't invest). It is then optimal to give the VC the entire revenue in case of failure, and to share R^u between the VC and the entrepreneur so that efforts are set as defined in proposition 3. Suppose now that the financier increases her investment to R^d. As before, to recoup his investment, she must be given R^d both in case of success and in case of failure. This has no impact on the efforts provided. However, it implies that the expected revenue of the VC decreases (although efforts remain constant, the VC's share of outcome in case of success and in case of failure decreases) and that her participation constraint is violated. To restore participation, the VC's share of outcome in case of success must increase, which leads to the result of corollary 2.

The next corollary summarizes the impact of the entrepreneur's wealth constraint on the value of the project and on the financial agreement.

Corollary 3 *The NPV of the project depends on the initial investment and on the wealth of the entrepreneur in the following way :*

- *if $I \in]K + R^d; \overline{I}]$, the project can be implemented with or without the financial participation of the entrepreneur. The NPV increases with the entrepreneur's investment.*
- *If $I \in]\overline{I}; I_{max}]$, the project cannot be implemented without the financial participation of the entrepreneur. The NPV increases with the entrepreneur's investment.*

I_{max} is a constant defined in appendix.

We already know from corollary 1 that when $I < K + R^d$, it is optimal to implement the project with outside financing only. Corollary 3 states that it is value-destructive to ask for more outside financing : when the investment I needed is greater than $K + R^d$, it is better to have the entrepreneur finance the complement rather than the outside investors. This is due to the fact that when outside financing increases above $K + R^d$, there is too much effort a and too little effort e being exerted

compared to the optimum. Last, recall from proposition 4 that there is a maximal amount that can be raised from outside investors (denoted $\overline{I}$ in the proposition) : if I is greater than $\overline{I}$, the project cannot be implemented without the financial participation of the entrepreneur.
It is interesting to investigate when the different cases mentionned above may happen. Note that all bounds mentionned in corollary 3 (that is, $K + R^d$,$\overline{I}$ and I_{max}) increase with $R^u - R^d$. $R^u - R^d$ can be interpreted as the potential profits to be gained when implementing the project[14]. I being small compared to $R^u - R^d$ reflects a project with high potential profitability. An implication of the model is that those types of projects are likely to be financed entirely by outside investors. As potential profitability decreases, the value of the project is likely to decrease, except if the entrepreneur can contribute to the financing of the investment.

The analysis of the optimal contract sheds light on some important features of VC financing. First, It provides a rationale for the commonly observed behavior of VC investors, or business angels. One of their characteristics is that they make use of their own competences along with their financial investment to help developping the projects they are backing. For instance, Gorman and Sahlman (1989) report that venture capitalists spend a great deal of time in the firms they invest in, providing advice and experience. Hellmann and Puri (2001) also document this "soft side" of venture capital. Less unanimity is found concerning the advising role of business angels. Although it is sometimes argued that they are less deeply involved in the projects they finance (see for instance Ehrlich, De Noble, Moore and Weaver (1994)), many authors do find an important advising role in angels financing[15]. Prowse (1998) reports from interviews with business angels that :

> Active angels almost always provide more than money. Angels will often help companies arrange additionnal financing, hire top management, and recruit knowledgeable board members. Angels also help solving major operational problems [...] and develop the company's long-term strategy.

[14] In this model, the actual expected profits are endogenous, since they depend on the levels of effort provided by the entrepreneur and the VC. $R^u - R^d$ represents the part of the NPV that is exogenous, this is why it can be lousely called "potential profits".

[15] Other evidence is found in Freear, Sohl and Wetzel (1994), or Mason and Harrison (2000). See also Berger and Udell (1998) and Lerner (1998) for a discussion on the different characteristics of angel investors.

This model explains why the joint provision of advice and money is so often observed in the case of start-ups. Although business expertise is not the exclusive property of VCs, it may sometimes be the only way for an entrepreneur to obtain efficient advice. Last, the model determines the optimal levels of investment of each party according to the potential profitability of the projects. The implications are that high potential projects are likely to be financed by outside investors only. Projects with lower prospects can also be financed by outside investors only, but benefit from the financial participation of the entrepreneur. For those projects, we expect the relation between the level of investment of the entrepreneur and the profitability of the project to be positive.

4 Implementing the optimal contract : optimal financial agreements for start-ups

This section investigates the question of the implementation of the optimal contracts described in the previous section. More precisely, the idea is to determine what kind of financial claims best cope with the double moral hazard problem at stake in this model. The objective is to contribute to the growing literature on the use of outside equity (or equity-like claims) by providing a rationale for the use of convertible and equity-like claims in the special setting of VC financing. We will restrict the analysis to the case where the only outside investor is the VC. As we saw before, such a restriction is harmless from the point of view of efficiency : proposition 3 states that the presence of a pure financier in the contract between the entrepreneur and the VC is irrelevant to the levels of effort achieved. It will however allow for a more precise identification of the optimal claims : as the entrepreneur and the VC share both the revenue in case of failure and in case of success, we will be able to compare how the shares given to each agent vary with the performance of the firm. The following proposition states which financial claims can implement the optimal contract, as a function of the level of outside financing.

Proposition 5 *There exists a threshold K', lying between K and $K + R^d$, such that :*

- *when $A_{VC} \in [K, K'[$, the optimal contract can be interpreted as common stocks given to the VC and preferred stocks given to the entrepreneur.*

- *When $A_{VC} = K'$, the optimal contract can be implemented by giving each agent common stocks.*
- *When $A_{VC} \in]K', \overline{I}]$, the optimal contract can be implemented by issuing preferred stocks or convertible bonds.*
- *Last, when $A_{VC} \in [K + R^d, \overline{I}]$, the firm can also be financed through straight debt.*

When the outside financing is rather small, the VC must be given higher powered incentives. In this case, the preferred stocks given to the entrepreneur are structured in such a way that, although the two shareholders receive a revenue in the bad state, preferred stocks give a higher dividend than common stocks. In the good state, the outcome is high enough so that the two types of stocks give the same return. As a consequence, the VC who owns only common stocks is proportionally better remunerated in state R^u than in state R^d, which gives her incentives to exert effort.
When $A_{VC} = K'$, it is optimal to give the VC and the entrepreneur common stocks, so that the two agents enjoy the same rate of return in any given final outcome.
Last, when $A_{VC} > K'$, the VC must be given a large share of profits in order to recoup her investment. As there is little left for the entrepreneur, he is less induced to effort, and needs a higher powered incentive scheme. When the VC is given convertible bonds or preferred stocks, she gets a larger share of the outcome in state R^d. The entrepreneur is then (proportionnally) better remunerated in state R^u, which makes him intensify his effort to increase the probability of state R^u occurring.

Proposition 5 also states that it is sometimes possible for the entrepreneur to be financed with straight debt. However, this leads to less profitable projects. Indeed, financing through debt arises when the outside financing is larger than its optimal level $(K + R^d)$, hence when i) the potential profitability of the project is quite low and ii) the entrepreneur is wealth-constrained. The entrepreneur's effort is then too small, but the participation constraint of the investor precludes allocating a larger share of outcome to the entrepreneur. It is then impossible to induce him to exert more effort. As a consequence, the maximal value of the project can only be reached when using equity-like claims, like convertible bonds or preferred stocks. Moreover, note that even when issuing straight debt is possible, convertible bonds or preferred stocks do just as well. To conclude, although pure debt financing is possible in this model, we are able to derive conditions under which convertible bonds do better, while the reverse is not true.

Many papers have stressed the intensive use of convertible claims (bonds or preferred stocks) in VC financing (see for instance Sahlman (1988, 1990) or Kaplan and Strömberg (2000)). By focusing on the necessary joint efforts of the entrepreneur and the VC to develop efficiently the project, the model presented here offers a possible explanation for the common use of this type of financial claims. Also, Fenn, Liang and Prowse (1998), in a paper comparing empirically the financial claims used by business angels and venture capitalists, stress some differences between the two types of investors. In their sample of 107 US firms of high-tech sectors (medical equipment and software industry), they find that business angel-backed firms obtained an average funding of US $ 1.5 million, while venture capital-backed firms obtained an average funding of US $ 12 million. In addition, 3/4 of the business angels' interventions consisted in common stock acquisitions, while 3/4 of the venture capitalists' interventions consisted in convertible claims acquisitions. This is consistent with the results stated in proposition 5 which stipulates that when the investor's financial participation is low, she gets common stocks, while she obtains convertible bonds or preferred stocks when her investment is high. A special feature of the model is that the two regimes described above can be related to the characteristics of the projects. Indeed, the case where the investor's financial participation is low corresponds to a project which initial investment is low compared to its expected cash-flows. The second regime where the investor's participation is high corresponds to a project which initial investment is high compared to its expected cash-flows and to the wealth of the entrepreneur.

Last, it is important to stress that the way the optimal contract can be implemented is not unique : for instance, in our model, convertible bonds can do just as well as preferred stocks, and both can be used indifferently. This indeterminacy is itself an important feature of real venture capital contracts. As noted by Kaplan and Strömberg (2000) :

> " while convertible securities are used most frequently, venture capitalists also implement the same set of rights using combinations of multiple classes of common stock and straight preferred stock".

What is important is how the cash-flow rights allocated to each party (entrepreneur and venture capitalist) vary with the firm's performance. On this issue, Kaplan and Strömberg (2000) find that VCs' cash-flow rights tend to decrease with the firm's performance, while the founder's cash-flow rights tend to increase with performance. This is consistent with the second regime described in proposition

5 where the VC's investment is high, and where she is given convertible bonds, while the entrepreneur is given common stocks. In this case, the VC's cash-flow rights are higher in case of low outcome than in case of high outcome, while the entrepreneur's rights increase with performance.

5 Conclusion

This paper focuses on one special feature of venture capital intervention in start-ups, namely the ability of venture capitalists to provide advice to the entrepreneurs they are backing. The model proposes an explanation for why entrepreneurs starting a business the success of which depends crucially on their personal intellectual investment may not be willing to hire consultants. The reason is that the entrepreneur has to give up a share of profits to incentivize a consultant and the efficiency gain brought in by the consultant is too small to compensate the loss in the entrepreneur's profit. This result arises because the consultant has a lower productivity than the entrepreneur and because the entrepreneur has the outside option to implement the project without the advisor, hence he can refuse to hire him. In order to improve the profitability of his project, the entrepreneur has no choice but to rely on a venture capitalist who will provide advice *and* money. This can explain why venture capital-backed firms exhibit greater revenue growth than other similar firms. Besides, the analysis of the optimal financial contract provides a rationale for the frequently observed use of financial claims like common stocks, preferred stocks, or convertible bonds in VC agreements. When the joint efforts of the entrepreneur and the investor contribute to improve the profitability of the project, the level of effort provided is related to the amount invested. When the investor's investment is low, high powered financial claims, that enhance her revenue in the upper states of nature, increase her incentives to exert effort, which improves the project's profitability. The results state that when the investor's participation is low, she must be given common stocks, while she obtains convertible bonds or preferred stocks when her investment is high. These results are consistent with the empirical findings of Fenn, Liang and Prowse (1998) and Kaplan and Strömberg (2000) .

Appendix

Proof of lemma 1

The levels of effort chosen by the entrepreneur and the investor, given by the FOCs of IC_E and IC_{VC}, are :

$$\begin{cases} e &= \max\left[0; \min[1; \frac{1}{\beta}\left(\alpha_E^u R^u - \alpha_E^d R^d\right)]\right], \\ \text{and} & \\ a &= \max\left[0; \min[1; \frac{1}{\gamma}\left(\alpha_A^u R^u - \alpha_A^d R^d\right)]\right]. \end{cases}$$

It is easy to see that a sufficient condition for $\min[1; \frac{1}{\beta}\left(\alpha_E^u R^u - \alpha_E^d R^d\right)] = \frac{1}{\beta}\left(\alpha_E^u R^u - \alpha_E^d R^d\right)$ is :

$$\frac{1}{\beta}R^u < 1,$$

which constitutes assumption $A.1$.

Note that under $A.1$, we also have $\frac{1}{\gamma}\left(\alpha_A^u)R^u - \alpha_A^d R^d\right) < 1$ as $\frac{1}{\gamma} < \frac{1}{\beta}$ by assumption.

It remains to show that :

$$\max\left[0; \frac{1}{\beta}\left(\alpha_E^u R^u - \alpha_E^d R^d\right)\right] = \frac{1}{\beta}\left(\alpha_E^u R^u - \alpha_E^d R^d\right), \tag{2}$$

and

$$\max\left[0; \frac{1}{\gamma}\left(\alpha_A^u)R^u - \alpha_A^d R^d\right)\right] = \frac{1}{\gamma}\left(\alpha_A^u)R^u - \alpha_A^d R^d\right). \tag{3}$$

• Equation (2) is equivalent to showing that, when $e = 0$, the entrepreneur is never willing to choose α_E^u and α_E^d such that $\frac{1}{\beta}\left(\alpha_E^u R^u - \alpha_E^d R^d\right) < 0$.

When $e = 0$, a must be strictly positive (otherwise the project cannot be implemented), hence it is given by : $a = \frac{1}{\gamma}\left(\alpha_A^u R^u - \alpha_A^d R^d\right)$. Also, it is easy to see that $(PC)_{VC}$ and $(PC)_F$ are binding. Replacing a, A_F and A_{VC} by their value, the program defined page 9 becomes :

$$\max_{\alpha_A^u, \alpha_A^d} \quad (R^u - R^d)\tfrac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d) - \tfrac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + R^d - I$$

$$\text{s.t.} \quad \alpha_E^d R^d - \alpha_E^u R^u \geq 0, \tag{4}$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u, \tag{5}$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d \tag{6}$$

Suppose equation (4) is binding. Then the solution of the program is : $\alpha_A^u R^u - \alpha_A^d R^d = R^u - R^d$. This means that, given that $e = 0$, effort a is equal to $\frac{1}{\gamma}(R^u - R^d)$, which corresponds to its first best value. Suppose now that equation (4) is not binding, that is $\alpha_E^d R^d - \alpha_E^u R^u = \epsilon$, $\epsilon > 0$. It is easy to see that the same solution can be attained, that is : $\alpha_A^u R^u - \alpha_A^d R^d = R^u - R^d$. This is because when $\alpha_E^d R^d > \alpha_E^u R^u$, the share of outcome given to the financier can adjust so that it is always possible to induce the first best level of effort a[16]. The value of the objective function is then:

$$\frac{1}{2\gamma}[R^u - R^d]^2 + R^d - I.$$

Hence, when $e = 0$, it is efficient for the entrepreneur to choose α_E^u and α_E^d such that $\alpha_E^u R^u - \alpha_E^d R^d = 0$. With no loss of generality, effort e is always given by the expression : $\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d)$.

- Equation (3) means showing that when $a = 0$, the entrepreneur never chooses α_A^u and α_A^d such that $\frac{1}{\gamma}\left(\alpha_A^u R^u - \alpha_A^d R^d\right) < 0$.

By the same reasoning as before, when $a = 0$, the program solved by the entrepreneur is :

$$\max_{\alpha_E^u, \alpha_E^d} \quad (R^u - R^d)\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) - \frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + R^d - I$$

$$\text{s.t.} \quad \alpha_A^d R^d - \alpha_A^u R^u \geq 0, \tag{7}$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u, \tag{8}$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d \tag{9}$$

Because of the presence of the pure financier, the same solution can be attained whether equation (7) is binding or not and is characterized by : $\alpha_E^u R^u - \alpha_E^d R^d = R^u - R^d$. Given that $a = 0$, effort e is set at its first best value, i.e. $e = \frac{1}{\beta}(R^u - R^d)$. The value of the objective function is then :

$$\frac{1}{2\beta}[R^u - R^d]^2 + R^d - I.$$

As a consequence, there is no loss of generality in saying that effort a is always given by the expression : $\frac{1}{\gamma}(\alpha_E^u R^u - \alpha_E^d R^d)$.

□

[16]Note that this wouldn't be true anymore if there was no pure financier. In that case, setting $\alpha_E^d R^d = \alpha_E^u R^u$ when $e = 0$ would be the only way to induce the first best level of effort a. Equation (4) would then have to be binding.

Proof of proposition 1.

The first step to establish the proposition is to show that lemma 1 still holds when we impose $A_{VC} = 0$ in the general program. The main difference with the case where A_{VC} can be optimally chosen is that (PC_{VC}) may not be binding.

• Suppose first that $e = 0$. As before, $a = \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)$ and (PC_F) is binding. The program solved by the entrepreneur can be written :

$$\max_{\alpha_A^u, \alpha_A^d} \quad \frac{1}{\gamma}\left[R^u - R^d - (\alpha_A^u R^u - \alpha_A^d R^d)\right](\alpha_A^u R^u - \alpha_A^d R^d) + R^d - I - \alpha_A^d R^d$$

$$\text{s.t.} \quad \alpha_E^d R^d - \alpha_E^u R^u \geq 0 \tag{10}$$

$$\frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + \alpha_A^d R^d \qquad (PC_{VC})$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u,$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d$$

It is straightforward to see that the optimal solution is to set $\alpha_A^d = 0$ and $\alpha_A^u R^u = \frac{1}{2}(R^u - R^d)$. For the reasons mentionned in the proof of lemma 1, this solution is feasible whether equation (10) is binding or not.

• Suppose next that $a = 0$. We then have : $e = \frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d)$ and (PC_F) is binding. (PC_{VC}) is written :

$$\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d)(\alpha_A^u R^u - \alpha_A^d R^d) + \alpha_A^d R^d \geq 0.$$

Check that if $\alpha_A^u R^u = \alpha_A^d R^d$, the solution of the program is : $\alpha_E^u R^u - \alpha_E^d R^d = R^u - R^d$, that is, the effort e is set at its first best level, given that $a = 0$. If $\alpha_A^u R^u < \alpha_A^d R^d$ (for instance, $\alpha_A^u R^u = \alpha_A^d R^d - \epsilon,\ \epsilon > 0$), it is not possible anymore to induce the first best level of effort e. Indeed, at the optimum, we have : $\alpha_E^u R^u - \alpha_E^d R^d = R^u - R^d + \epsilon$ which implies that the level of effort e is too large compared to the optimum and the value of the objective function is strictly lower than in the case where $\alpha_A^u R^u = \alpha_A^d R^d$. Hence, lemma 1 still holds when there is no financial participation of the advisor.

The second step of the proof consists in solving the general program after replacing (IC_{VC}) and (IC_E) using the expressions in lemma 1. Note that (PC_F) is still binding and can also be replaced.

After manipulations, the program to solve is the following.

$$\max_{\alpha_E^\theta,\alpha_A^\theta,\theta\in\{u,d\}} [\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)][R^u - R^d - (\alpha_A^u R^u - \alpha_A^d R^d)]$$

$$-\frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + R^d - I - \alpha_A^d R^d$$

$$\text{s.t.} \quad \frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d) + \frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d)(\alpha_A^u R^u - \alpha_A^d R^d) + \alpha_A^d R^d \geq 0 \qquad (PC_{VC})$$

$$\alpha_E^u R^u + \alpha_A^u R^u \leq R^u,$$

$$\alpha_E^d R^d + \alpha_A^d R^d \leq R^d$$

Note that (PC_{VC}) cannot be binding if $e > 0$ and $a > 0$. (PC_{VC}) can only be binding if $a = 0$ and $\alpha_A^u = \alpha_A^d = 0$, which corresponds to the case where the entrepreneur doesn't hire a consultant. To establish proposition 1, it will be demonstrated that the entrepreneur is strictly better off if (PC_{VC}) is binding.

Setting $\alpha_A^d = 0$ is optimal since it lowers the expected outcome of the advisor, and increases the entrepreneur's profit without affecting the latter's incentives to effort. Define $X = \alpha_E^u R^u - \alpha_E^d R^d$ and $Y = \alpha_A^u R^u$. (PC_{VC}) can be rewritten : $\frac{1}{2\gamma}Y^2 + \frac{1}{\beta}XY \geq 0$. As $X > 0$, (PC_{VC}) is automatically satisfied when $Y \geq 0$ which implies that it is redondant compared to the feasibility constraint. The program solved by the entrepreneur is :

$$\max_{X,Y} \quad -\frac{1}{2\beta}X^2 + (\frac{1}{\beta}X + \frac{1}{\gamma}Y)(R^u - R^d - Y)$$

$$\text{s.t.} \qquad Y \geq 0$$

The objective function is concave if $2\beta > \gamma$ and convex otherwise. The lagrangian of the program is :

$$L = -\frac{1}{2\beta}X^2 + (\frac{1}{\beta}X + \frac{1}{\gamma}Y)(R^u - R^d - Y) + \lambda Y.$$

The solutions must verify :

$$\frac{\partial L}{\partial X} = 0 \quad \Leftrightarrow \quad -X + (R^u - R^d - Y) = 0 \qquad (11)$$

$$\frac{\partial L}{\partial Y} = 0 \quad \Leftrightarrow \quad -\frac{1}{\beta}X + \frac{1}{\gamma}(R^u - R^d - 2Y) = 0 \qquad (12)$$

$$\lambda \geq 0, \quad Y \geq 0, \quad \lambda Y = 0$$

If $\lambda = 0$, equations (11) and (12) imply $Y = \frac{\gamma-\beta}{\gamma-2\beta}(R^u - R^d)$. Note however that this solution is not feasible if $2\beta > \gamma$ (since Y must be positive). In that case, we must have $Y = 0$ and $X = R^u - R^d$.

If $2\beta < \gamma$, $Y = \frac{\gamma-\beta}{\gamma-2\beta}(R^u - R^d)$ is feasible but recall that in that case, the objective function is convex, which means that Y defined above is a minimum. The maximum is then also defined by $Y = 0$ and $X = R^u - R^d$. To conclude, it is optimal for the entrepreneur to set $Y = 0$, that is not to hire a consultant. The optimal level of effort of the entrepreneur is then : $e = \frac{1}{\beta}(R^u - R^d) = e^{FB}$. Note that if $e = e^{FB}$, the expected outcome of the pure financier is at most equal to R^d, which means that this solution holds for $A_F \leq R^d$. In case the entrepreneur needs to borrow more than R^d (say, in case he is wealth-constrained), it can be shown (using the same methodology) that the result of the proposition goes through : the entrepreneur never hires a consultant. However, because outside financing is too large, he is induced to exert a level of effort strictly lower than the first best. More formal proof is available upon request.

□

Proof of proposition 2

Using lemma 1, the program of the entrepreneur becomes :

$$\max_{\alpha_E^\theta, \alpha_A^\theta, \theta\in\{u,d\}, A_{VC}, A_F} \frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)(\alpha_E^u R^u - \alpha_E^d R^d) + \alpha_E^d R^d - (I - (A_{VC} + A_F))$$

$$\text{s.t. } \frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + \frac{1}{\beta}(\alpha_A^u R^u - \alpha_A^d R^d)(\alpha_E^u R^u - \alpha_E^d R^d) + \alpha_A^d)R^d \geq A_{VC} \qquad (PC)_{VC},$$

$$\left(\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)\right)\left(R^u - R^d - (\alpha_E^u R^u - \alpha_E^d R^d) - (\alpha_A^u R^u - \alpha_A^d R^d)\right)$$

$$+R^d - (\alpha_A^d R^d + \alpha_E^d R^d) \geq A_F \qquad (PC)_F$$

$$(\alpha_E^u, \alpha_E^d, \alpha_A^u, \alpha_A^d) \geq 0$$

$$1 - (\alpha_E^u + \alpha_A^u) \geq 0$$

$$1 - (\alpha_E^d + \alpha_A^d) \geq 0$$

Replacing A_F and A_{VC} in the objective function using the fact that the participation constraints are binding gives the following program :

$$\max_{\alpha_E^\theta, \alpha_A^\theta, \theta\in\{u,d\}} -\frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + (R^u - R^d)[\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)]$$

$$-\frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + R^d - I$$

$$\text{s.t. } \alpha_A^\theta \geq 0;\ \alpha_E^\theta \geq 0,\ \theta \in \{u,d\};\ 1 - (\alpha_A^u + \alpha_E^u) \geq 0;\ 1 - (\alpha_A^d + \alpha_E^d) \geq 0$$

Consider first not taking into account the feasibility constraints, and define $X = \alpha_E^u R^u - \alpha_E^d R^d$ and $Y = \alpha_A^u R^u - \alpha_A^d R^d$. The objective function is concave since the hessian is negative semidefinitive. First order conditions of the maximisation of the objective function give :

$$X = Y = R^u - R^d.$$

It is straightforward to see that if feasible, this solution corresponds to the first best levels of effort being exerted. Replacing X and Y by their value, and using the fact that $\alpha_E^u + \alpha_A^u \leq 1$, it follows that this solution is feasible iff :

$$2(R^u - R^d) + \alpha_E^d R^d + \alpha_A^d R^d \leq R^u.$$

Since the smallest possible value for α_E^d and α_A^d is 0, it follows that first best levels of effort can be implemented iff : $R^u \leq 2R^d$, which completes the proof of the first part of the proposition.

When $R^u > 2R^d$, one must write down the lagrangian L of the program, including all the feasibility constraints described above :

$$\begin{aligned} L = -\frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + (R^u - R^d)[\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)] \\ -\frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + \lambda_1 \alpha_E^u R^u + \lambda_2 \alpha_E^d R^d + \lambda_3 \alpha_A^u R^u + \lambda_4 \alpha_A^d R^d \\ +\lambda_5 (R^u - (\alpha_E^u R^u + \alpha_A^u R^u)) + \lambda_6 (R^d - (\alpha_E^d R^d + \alpha_A^d R^d)) \end{aligned}$$

Straight application of the theorem of Kuhn-Tucker and tedious algebra give the following solution :

$$\begin{cases} \alpha_E^{d*} R^d &= \alpha_A^{d*} R^d = 0, \\ \alpha_E^{u*} R^u &= \frac{\gamma R^d + \beta(R^u - R^d)}{\gamma + \beta}, \\ \alpha_A^{u*} R^u &= \frac{\beta R^d + \gamma(R^u - R^d)}{\gamma + \beta}. \end{cases}$$

□

Proof of proposition 3.

The program to be solved is the same as in the previous section, except that the non decreasing revenue constraint must be added, i.e. :

$$R^u - (\alpha_E^u + \alpha_A^u) R^u \geq R^d - (\alpha_E^d + \alpha_A^d) R^d. \tag{13}$$

Note that the constraint $\alpha_E^u + \alpha_A^u \leq 1$ becomes redundant as it is automatically satisfied when equation (13) holds. The new lagrangian is the following :

$$L = -\frac{1}{2\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)^2 + (R^u - R^d)[\frac{1}{\beta}(\alpha_E^u R^u - \alpha_E^d R^d) + \frac{1}{\gamma}(\alpha_A^u R^u - \alpha_A^d R^d)]$$
$$-\frac{1}{2\beta}(\alpha_E^u R^u - \alpha_E^d R^d)^2 + \lambda_1 \alpha_E^u R^u + \lambda_2 \alpha_E^d R^d + \lambda_3 \alpha_A^u R^u + \lambda_4 \alpha_A^d R^d$$
$$+\lambda_5(R^u - R^d - (\alpha_E^u R^u - \alpha_E^d R^d) - (\alpha_A^u R^u - \alpha_A^d R^d)) + \lambda_6(R^d - (\alpha_E^d R^d + \alpha_A^d R^d))$$

Again, straight application of the theorem of Kuhn-Tucker gives :

$$\begin{aligned} \alpha_E^{u*} R^u - \alpha_E^{d*} R^d &= \tfrac{\gamma}{\gamma+\beta}(R^u - R^d), \\ \alpha_A^{u*} R^u - \alpha_A^{d*} R^d &= \tfrac{\beta}{\gamma+\beta}(R^u - R^d). \end{aligned}$$

Note that $\alpha_E^u R^u + \alpha_A^u R^u$ is at least equal to $R^u - R^d$ which implies that at most, the revenue of the pure financier is equal to R^d in state u. As the constraint (13) is binding, it follows that $A_F^* \leq R^d$.

□

Proof of corollary 1.

Replacing α_E^u, α_A^u, α_E^d and α_A^d using the solutions described in proposition 3 in (PC_F) and (PC_{VC}) gives :

$$\begin{aligned} A_F^* &= R^d - \alpha_E^d R^d - \alpha_A^d R^d \\ A_{VC}^* &= \frac{(R^u - R^d)^2}{(\gamma+\beta)^2}\left(\frac{\beta^2 + 2\gamma^2}{2\gamma}\right) + \alpha_A^d R^d \end{aligned}$$

Define $K \equiv \frac{(R^u - R^d)^2}{(\gamma+\beta)^2}\left(\frac{\beta^2+2\gamma^2}{2\gamma}\right)$, and the first part of the corollary follows immediately. Recall that the expression of the value of project, V, is given by equation (1) page 6. Compute V^* using lemma 1 and proposition 3 to establish the remaining of the corollary (note that V^* is a constant since the levels of effort derived in proposition 3 are constant).

□

Proof of proposition 4.

Corollary 1 states that at the optimum defined by proposition 3, the amount of outside financing is no greater than $K + R^d$. This implies that if $I > K + R^d$, the entrepreneur must complement the contributions of the VC and the financier to attain the (second best) outcome. If he is wealth-constrained, one must solve the general program adding the constraint :

$$A_{VC} + A_F > K + R^d. \tag{14}$$

Replace A_F and A_{VC} by their value in (PC_{VC}) and (PC_F), set $\alpha_E^d = 0$ (which is obviously optimal when equation (14) holds) and use the fact that constraint (13) is binding to get :

$$A_{VC} + A_F = -\left(\frac{1}{\beta} - \frac{1}{2\gamma}\right)Y^2 + \frac{1}{\beta}(R^u - R^d)Y + R^d, \tag{15}$$

where Y stands for $\alpha_A^u R^u - \alpha_A^d R^d$. The determinant Δ is :

$$\Delta = \frac{(R^u - R^d)^2}{\beta^2} - 2((A_{VC} + A_F) - R^d)\frac{2\gamma - \beta}{\gamma\beta},$$

which is positive iff :

$$A_{VC} + A_F \leq R^d + \frac{\gamma}{2\beta}\frac{(R^u - R^d)^2}{2\gamma - \beta} \equiv \overline{I}.$$

Hence the maximal amount of outside financing is $\overline{I}$. The solution is readily computed and gives :

$$Y = \frac{\gamma(R^u - R^d) - \gamma\beta\sqrt{\Delta}}{2\gamma - \beta}.$$

Replacing Y by its value, and using equation (13) to find the expression of α_E^u gives, for $A_{VC} + A_F > K + R^d$:

$$\begin{aligned}
\alpha_E^{d*} &= 0, \\
\alpha_E^{u*} &= \tfrac{(\gamma - \beta)(R^u - R^d) + \gamma\beta\sqrt{\Delta}}{2\gamma - \beta}, \\
\alpha_A^{u*} - \alpha_A^{d*} &= \tfrac{\gamma(R^u - R^d) - \gamma\beta\sqrt{\Delta}}{2\gamma - \beta}.
\end{aligned}$$

□

Proof of corollary 2.

Use lemma 1 and proposition 4 to compute the optimal levels of effort when $I \geq K + R^d$ and the entrepreneur is wealth-constrained. It follows immediately that e decreases with I and a increases with I.

□

Proof of corollary 3.

I_{max} is simply the maximum possible level of initial investment (i.e. such that the NPV V is equal to zero). Replacing A_{VC} and A_F by their value in $(PC)_{VC}$ and PC_F into the objective function, and using the fact that equation (13) is binding gives the following expression of the NPV of the project :

$$V = -\frac{\gamma+\beta}{2\beta\gamma}Y^2 + \frac{1}{\gamma}(R^u - R^d)Y + \frac{1}{2\beta}(R^u - R^d)^2 + R^d - I,$$

where Y stands for $\alpha_A^u R^u - \alpha_A^d R^d$. Computing the determinant of the RHS of the equation of V, it is easy to see that V can only be positive if :

$$I \leq R^d + \frac{\gamma^2+\beta^2+\gamma\beta}{2\gamma\beta(\gamma+\beta)}(R^u - R^d)^2 \equiv I_{max}.$$

Next, the argument of corollary 3 goes as follows :

- if $I \in]K + R^d; \overline{I}]$, the higher the investment of the entrepreneur, the lower the extra outside funding needed, thus the higher the level of effort of the entrepreneur, and the lower the level of effort of the VC (corollary 2) : at best, the entrepreneur finances $I - (K + R^d)$ and the NPV is maximal as stated in corollary 1.
- if $I \in]\overline{I}; I_{max}]$, then the entrepreneur *must* participate financially so that the project can be undertaken (recall that $\overline{I}$ is the maximum amount of outside financing).

□

Proof of proposition 5.

- Common stocks for the investor, preferred stocks for the entrepreneur.

Preferred stocks ensure a minimum rate of return to their owner (or minimum dividend), before common stocks' returns are paid. When the outcome of the project is sufficiently high, both types of stocks give the same rate of return. Let $\underline{R}$ be the minimum revenue required by the preferred stocks' owner. $\underline{R}$ represents the minimum dividend pledged on each preferred stock, multiplied by the number of preferred stocks. Let α be the quantity of preferred stocks in the firm's equity. $(1-\alpha)$ is the fraction of common stocks. To be able to distinguish between preferred and common stocks, assume that $\alpha R^d < \underline{R} \leq R^d$ and $\underline{R} < \alpha R^u$. Hence, when the outcome of the project is low, it is impossible to remunerate common stocks with the same dividend as preferred stocks. When the outcome is high, both types of stocks generate the same dividend. Under these assumptions, the optimal contract can be implemented by giving common stocks to the investor and preferred stocks to the entrepreneur iff :

$$\begin{aligned}
(1-\alpha_E^d)R^d &= R^d - \underline{R}, && (16)\\
(1-\alpha_E^u)R^u &= (1-\alpha)R^u, && (17)\\
\alpha &\in \left]\frac{\underline{R}}{R^u}, \frac{\underline{R}}{R^d}\right[, && (18)\\
\underline{R} &\leq R^d. && (19)
\end{aligned}$$

When $A_{VC} \in [K\,,\,K+R^d[$, (16) and (17) are written :

$$\begin{aligned}
\underline{R} &= R^d + K - A_{VC},\\
\alpha &= \frac{R^u - \frac{\beta}{\beta+\gamma}(R^u - R^d) - A_{VC} + K}{R^u}.
\end{aligned}$$

It is easy to check that (19) is satisfied iff $A_{VC} \geq K$. Besides, (18) is satisfied iff :

$$A_{VC} < K + \frac{\beta}{\beta+\gamma}R^d \equiv K'.$$

• Common stocks for both agents.

Let α be the fraction of the firm's equity attributed to the entrepreneur, and $(1-\alpha)$ the fraction given to the investor. The contract can be implemented by common stock issuing iff :

$$\begin{aligned}
(1-\alpha_E^d)R^d &= (1-\alpha)R^d,\\
(1-\alpha_E^u)R^u &= (1-\alpha)R^u,\\
\alpha &\in \left]0,1\right[.
\end{aligned}$$

When $A_{VC} \in [K\,,\,K+R^d[$, such a contract is possible iff :

$$A_{VC} = K'.$$

• Convertible bonds or preferred stocks issuing.

In our simplistic model, issuing convertible bonds or preferred stocks generates the same pattern of return for their owner. As long as preferred stocks ensure a minimum dividend to be given to their owner, they are equivalent to convertible bonds, where the face value of the bond corresponds to a minimum dividend pledged before common shareholders are remunerated. Moreover, when the outcome of the project is high, bonds are converted, and the return they generate is similar to preferred (or common) stocks. Differences between these two types of claim usually concern the right to trigger bankruptcy (that convertible bond holders have, while preferred stock holders don't have), as well as the additive feature of preferred stocks dividends. In our model, those differences don't exist, since the firm only lives for one period. Hence convertible bonds (or preferred stocks) will be characterized by a face value D (or a minimum global dividend $\underline{R}$), and a fraction $1-\alpha$ of the firm's equity, such that :

- if $(1-\alpha)R^\theta \leq D$ $(\theta \in \{d;u\})$, the investor gets $\min[D;R^\theta]$;
- if $(1-\alpha)R^\theta > D$, the investor gets $(1-\alpha)R^\theta$.

To be able to distinguish between convertible bonds and common stocks, it is necessary to assume $(1-\alpha)R^d < D < (1-\alpha)R^u$[17].

1) Suppose first that $D < R^d$.

The contract can be implemented by convertible bonds issuing iff :

$$(1-\alpha_E^d)R^d = D, \quad (20)$$

$$(1-\alpha_E^u)R^u = (1-\alpha)R^u, \quad (21)$$

$$\alpha \in \left]\frac{R^d-D}{R^d}, \frac{R^u-D}{R^u}\right[, \quad (22)$$

$$D < R^d. \quad (23)$$

Such a contract is only possible if $A_{VC} \in [K, K+R^d[$, since the investor's revenue must be strictly lower than R^d in state d. Replacing α_E^d and α_E^u by their values, (20) and (21) become :

$$D = A_{VC} - K,$$

[17]The previous analysis has shown that when the investor converts in both states of nature, such a contract is equivalent to common stocks issuing, and is only possible for $I - A = K'$.

$$1-\alpha = \frac{1}{R^u}\left[\frac{\beta}{\beta+\gamma}(R^u - R^d) + A_{VC} - K\right].$$

Condition (23) is satisfied iff $A_{VC} < K + R^d$. Moreover, condition (22) implies : $A_{VC} > K'$. It follows that convertible bonds issuing (as structured above) is possible iff $A_{VC} \in]K', K + R^d[$.

2) Suppose now that $D \geq R^d$.

Such a contract exists only if : $A_{VC} \geq K + R^d$, since the investor's revenue must be equal to R^d in state d. Financing through convertible bonds is possible iff :

$$\begin{aligned}
(1-\alpha_E^d)R^d &= R^d, \\
(1-\alpha_E^u)R^u &= \max[D; (1-\alpha)R^u], \\
D &\geq R^d.
\end{aligned}$$

Such a contract is always possible when $A_{VC} \geq K + R^d$. Indeed, in this interval, $\alpha_E^d = 0$, which implies that the first condition above is satisfied. Moreover, converting in state u implies $R^d \leq (1-\alpha_E^u)R^u$, which is verified. Last, if there is no conversion, it is equivalent to issue convertible bonds or straight debt, which completes the proof of proposition 5.

□

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